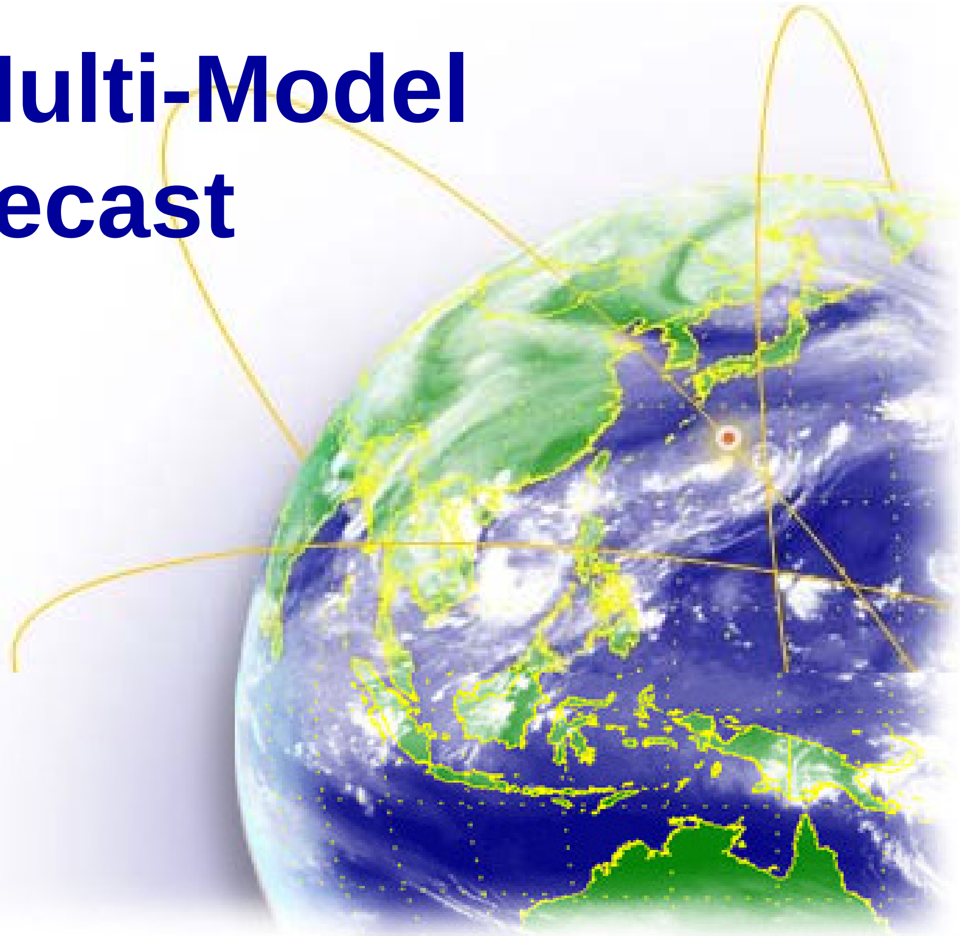
Lecture

Introduction to Multi-Model Probabilistic Forecast

Saji N. Hameed



Content

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2. Empirical Probability Distribution
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4. Gamma Probability Distribution
5. Multi-Model Ensemble

CATEGORICAL PROBABILISTIC FORECAST

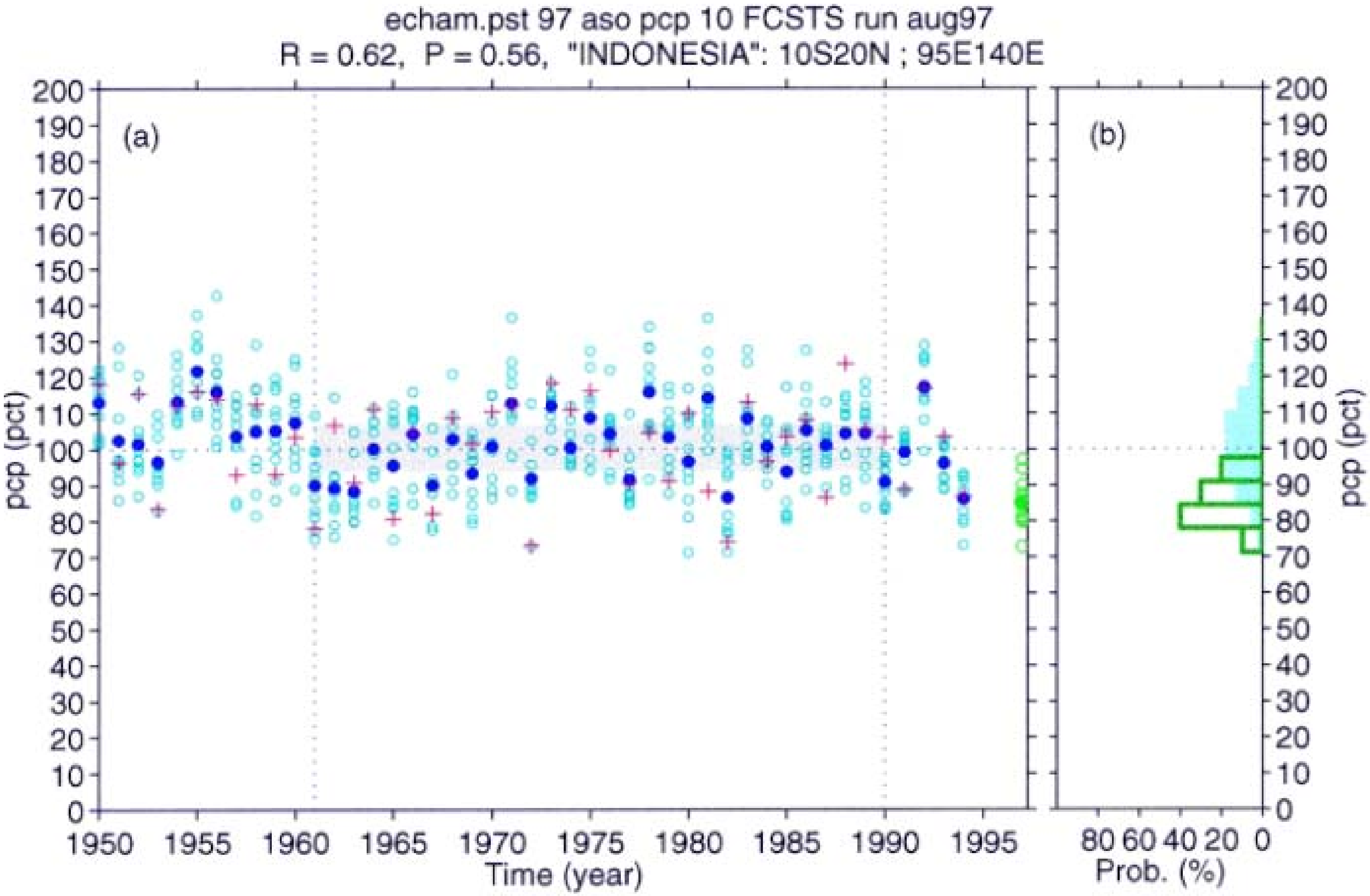
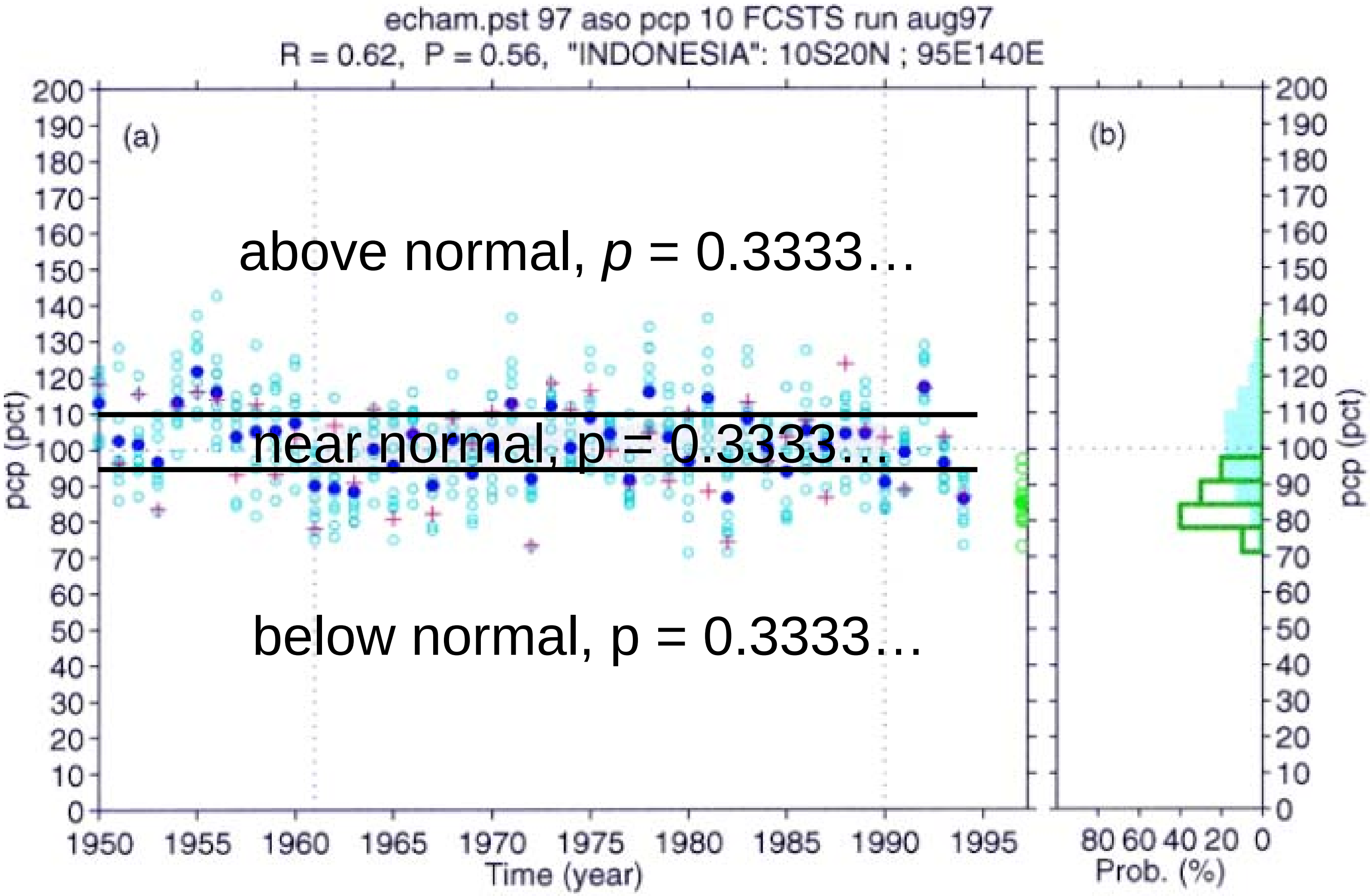


FIG. 6. (a) Historical performance of the ECHAM3 model for the August–October season compared to observations averaged over the “Indonesia” region (10˚S–20˚N, 95˚– 140˚E). The open blue circles show the model anomaly for individual ensemble members (expressed as a percentage of long-term mean), solid blue circles show the ensemble mean, and red crosses indicate the observed anomaly. The green circles are for the current forecast, and the solid green circle represents the ensemble mean. The gray-shaded area indicates the range of the near-normal tercile based on the climatological period 1961–90. The numbers at the top of the graph indicate the correlation between the ensemble mean simulation and the observed anomalies (*R*) and the tercile hit score (*P*). (b) Distribution of forecast members for August–October 1997 (open green bars) relative to the climatological distribution (solid blue bars). From Mason et al., 1999

CATEGORICAL PROBABILISTIC FORECAST



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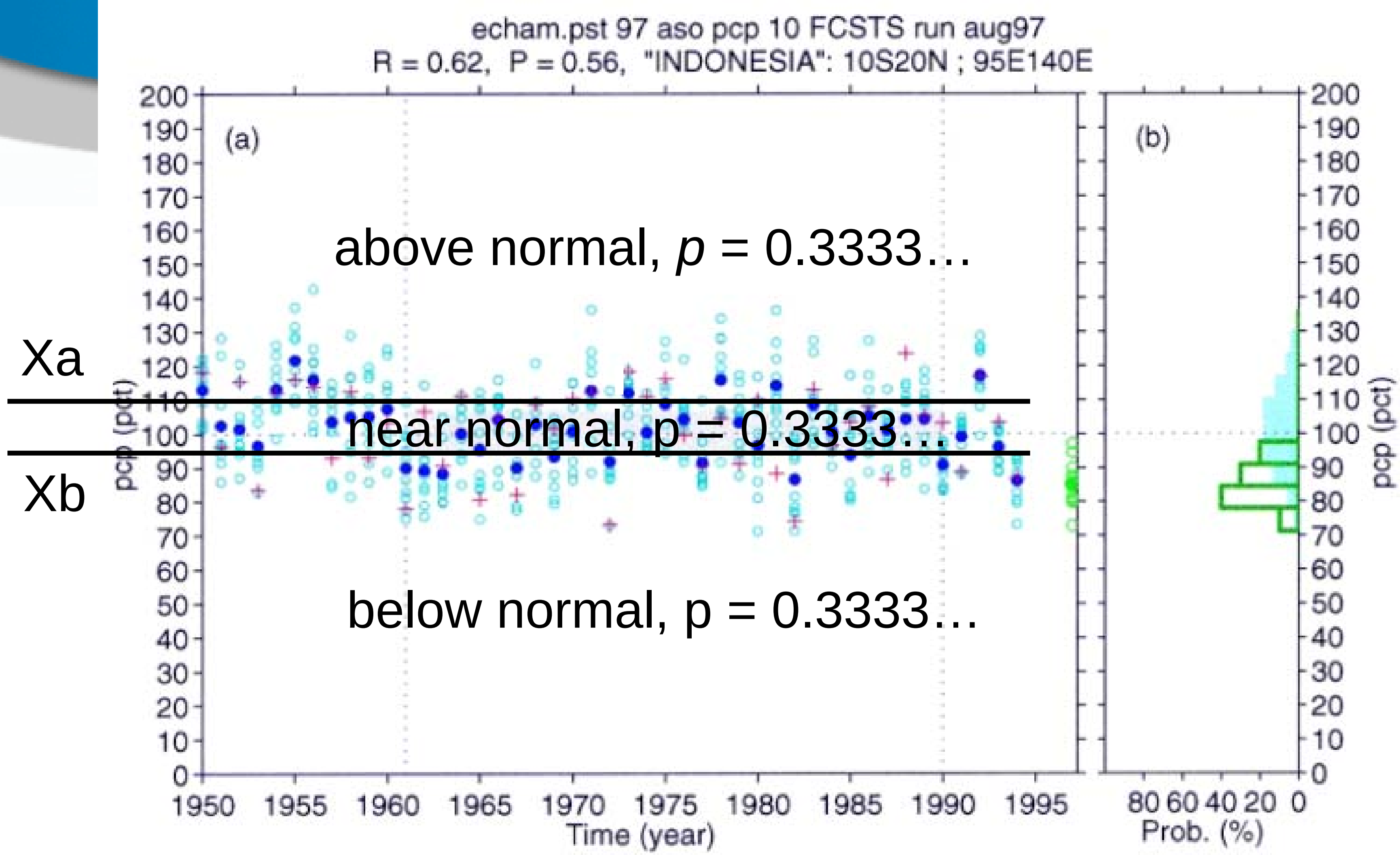

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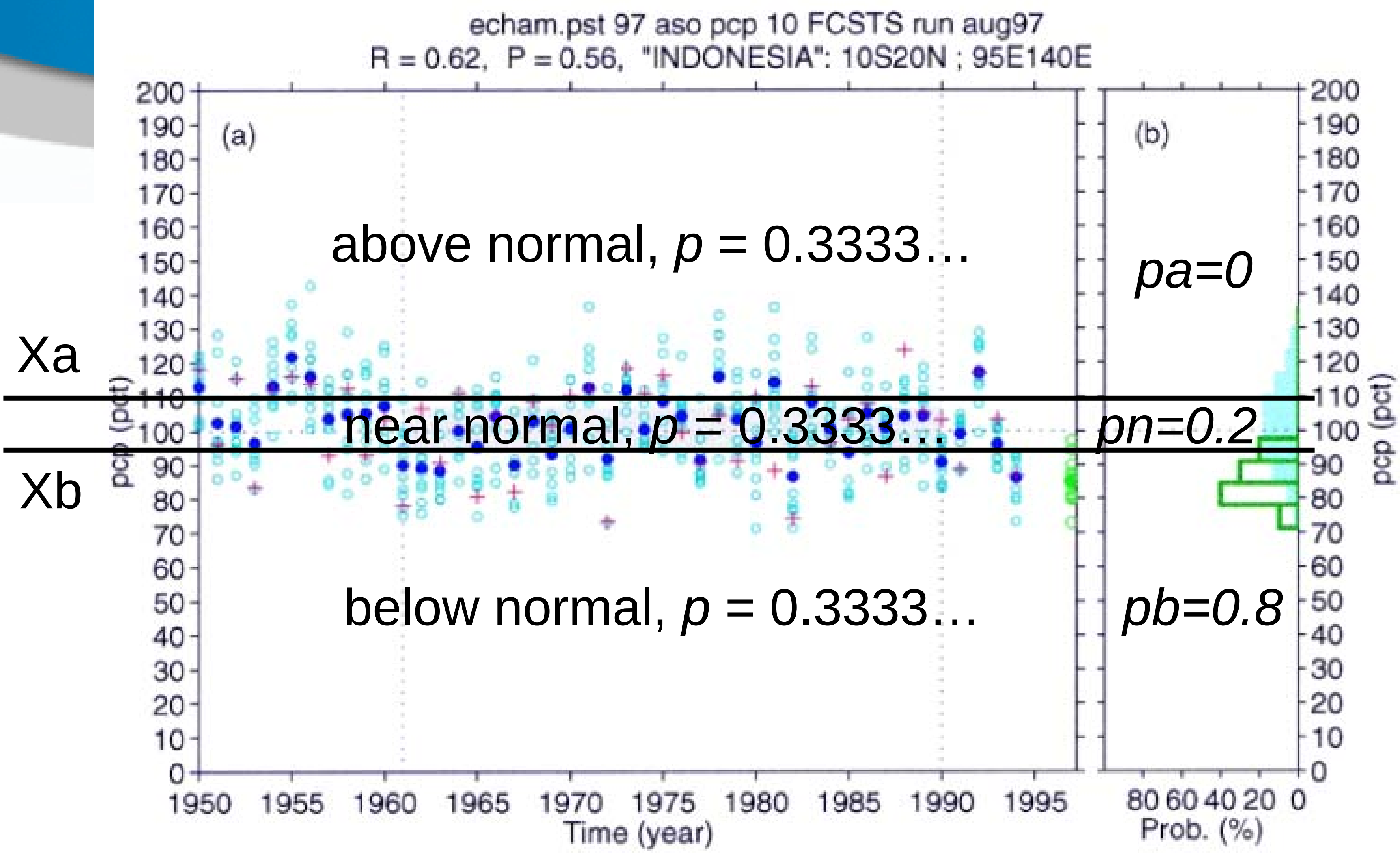

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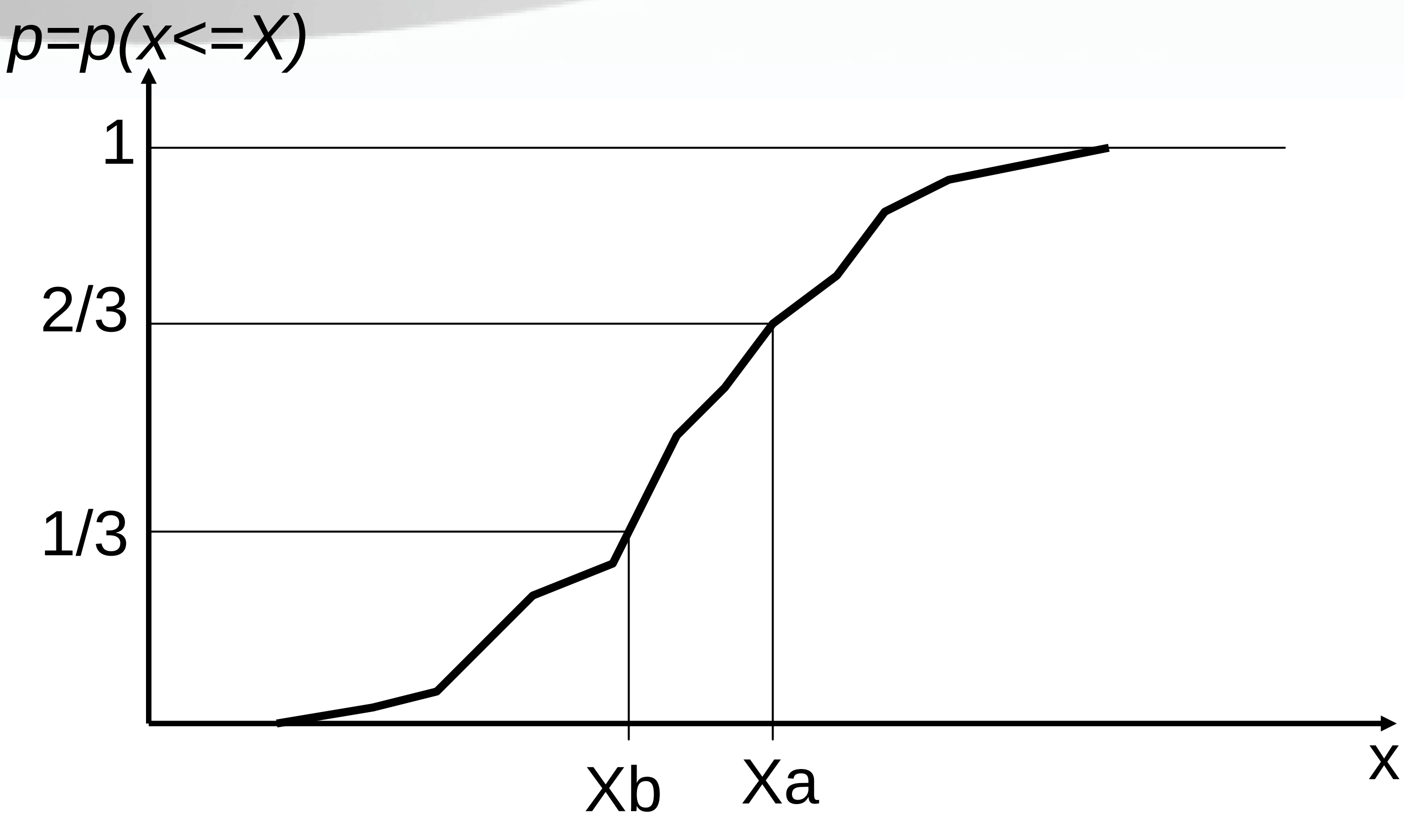
EMPIRICAL PROBABILITY DISTRIBUTION

What is the meaning of Xa and Xb?

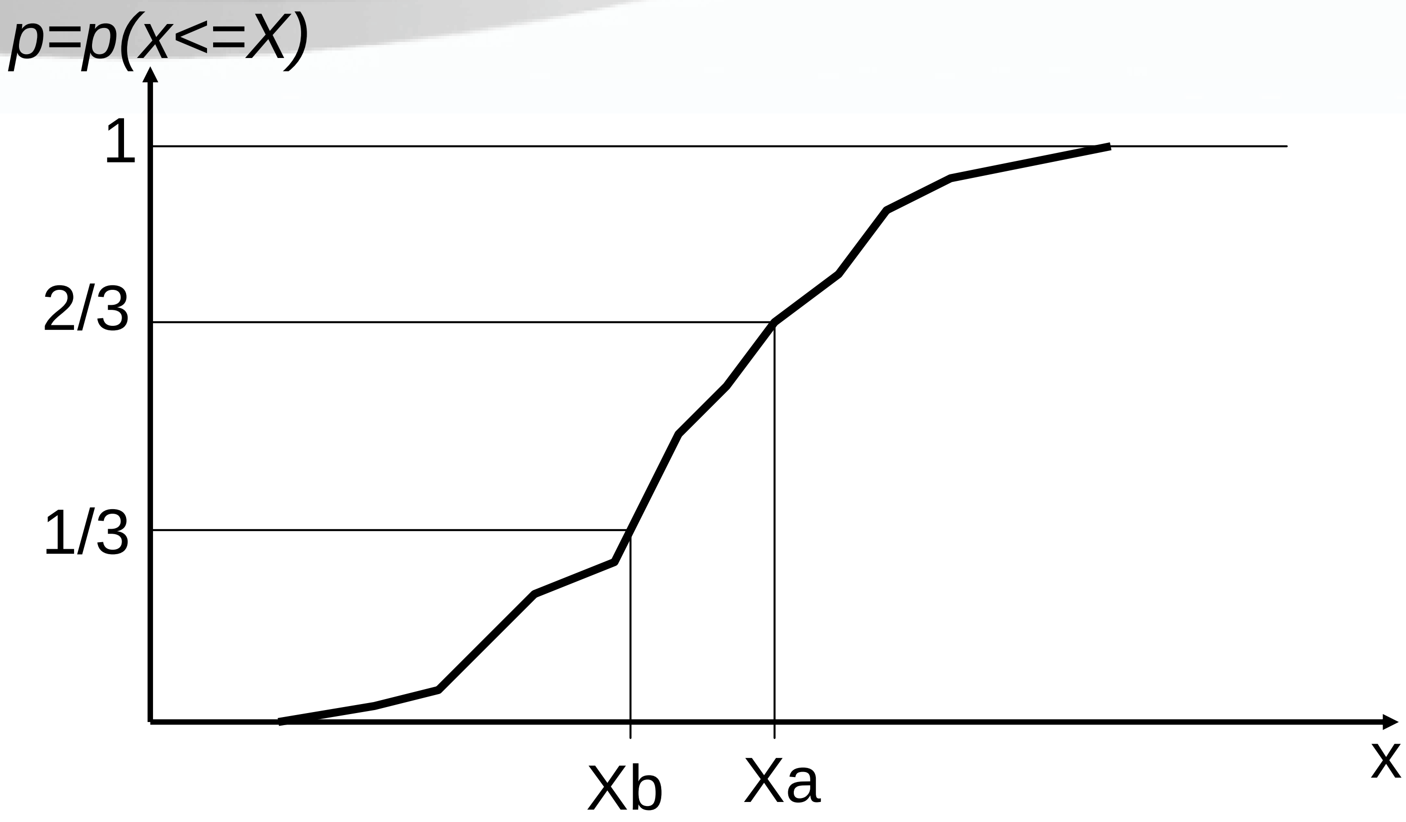


$p(x <= Xb) = 1/3$

$p(x <= Xa) = 2/3$

EMPIRICAL PROBABILITY DISTRIBUTION

How to estimate Xa and Xb?



If **xr** is ranked **x** than Xb = $xr_{i=N/3}$ while Xa = $xr_{i=2N/3}$

where N is HINDCAST sample size

EMPIRICAL PROBABILITY DISTRIBUTION

How to estimate probabilities of the categories below, near, above normal?

pb = mb/n

pn = mn/n

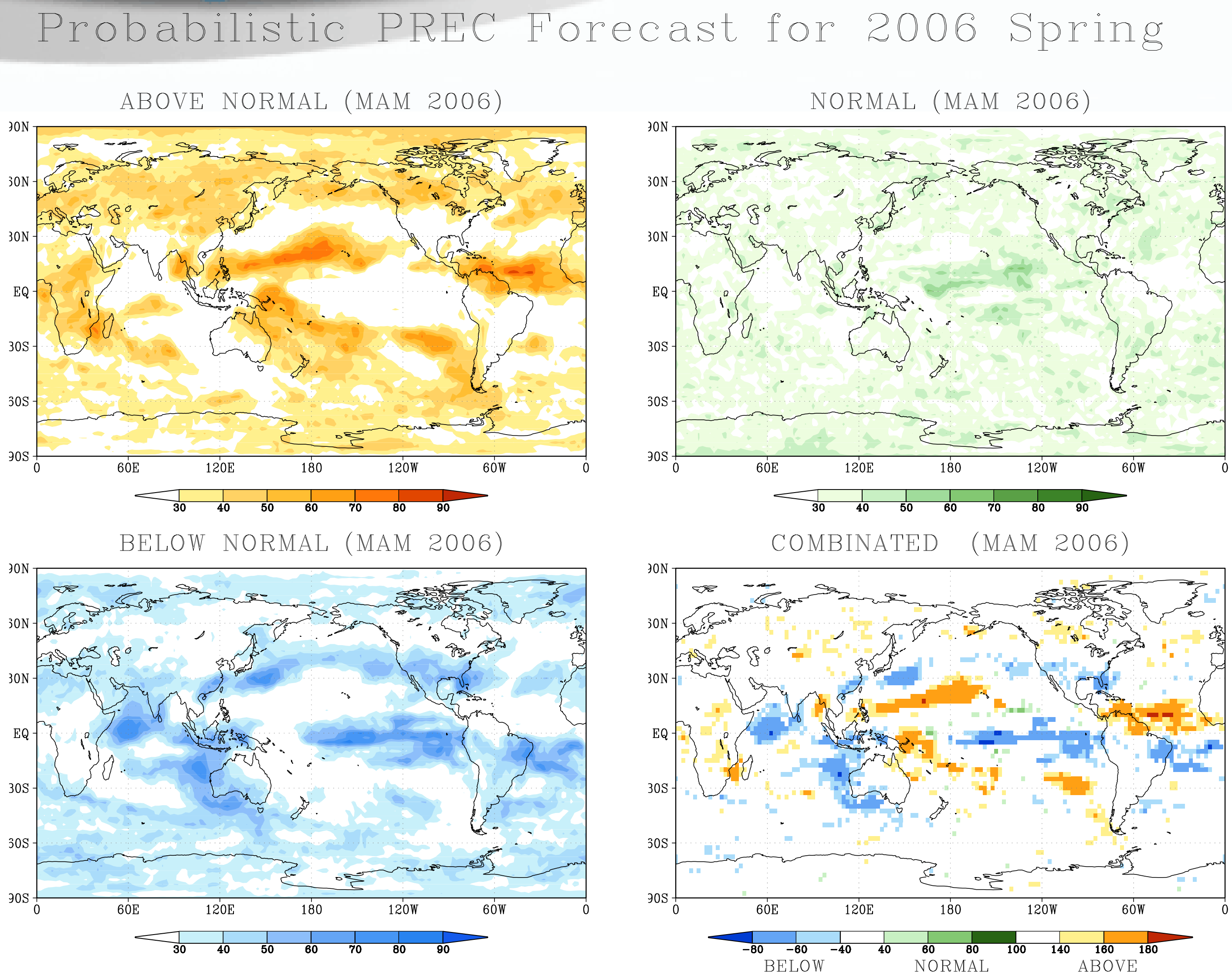
pa = ma/n

where n is FORECAST sample size;
mb, mn, ma – numbers of FORECAST ensemble members x (frequency) such that

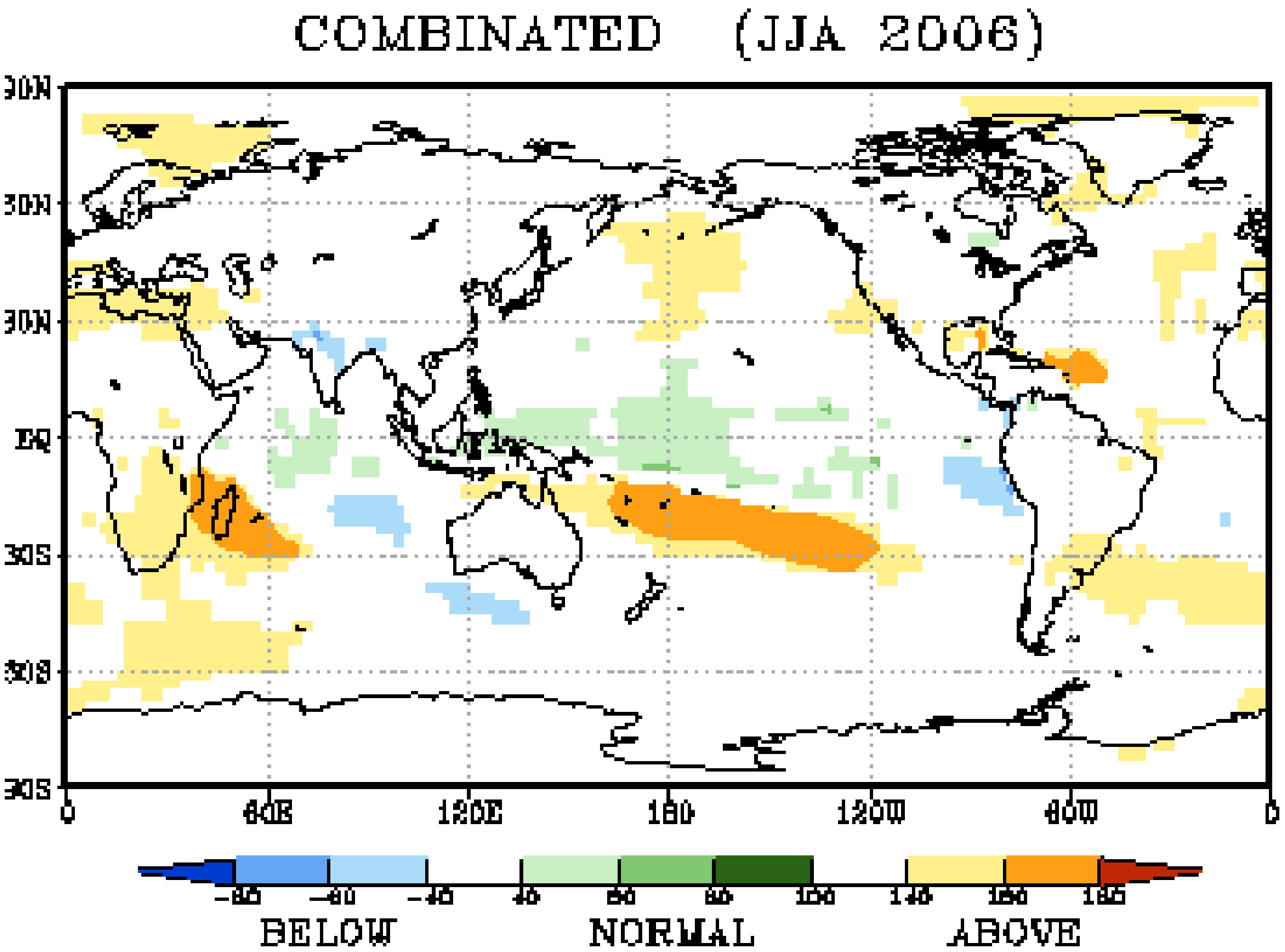
x <= Xb	Xb < x <= Xa	Xa < x
mb	mn	ma

EMPIRICAL PROBABILITY DISTRIBUTION

How does probabilistic forecast look like?



EMPIRICAL PROBABILITY DISTRIBUTION



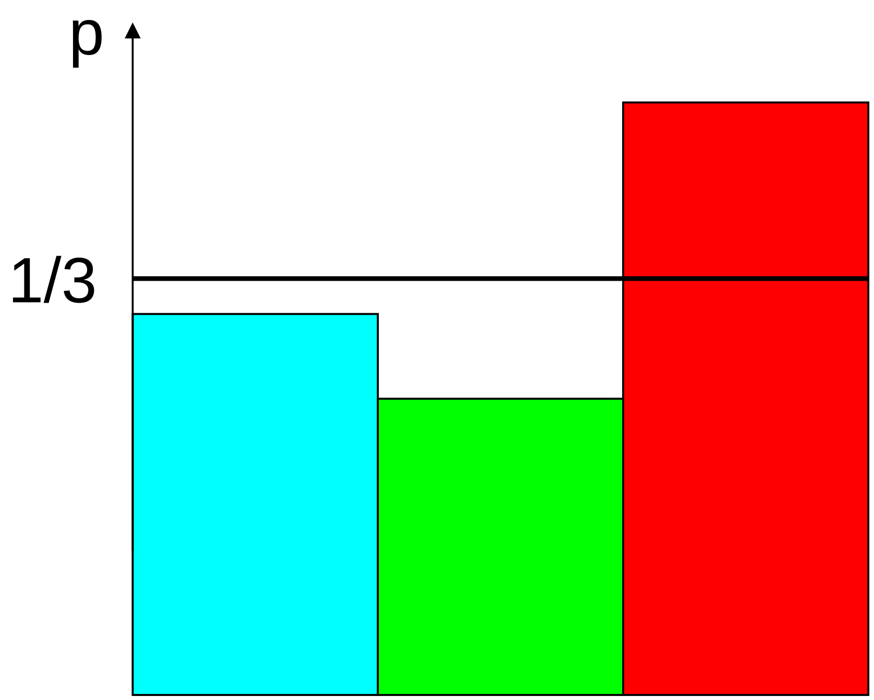
What does the fourth map mean?

Why is it a four-color map?

What does the white color mean?

EMPIRICAL PROBABILITY DISTRIBUTION

How to treat such a forecast?



EMPIRICAL PROBABILITY DISTRIBUTION

How to combine categories?

Binomial distribution:

$$\beta = \sum_{i=0}^{m-1} C_n^i p^i (1-p)^{n-i}$$

n – forecast sample size
m – number of ensemble members in largest category
p – theoretical probability

EMPIRICAL PROBABILITY DISTRIBUTION

How to combine categories?

χ^2 distribution

$$\chi^2 = n\sum_{i=1}^{k}\frac{(pe-po)^2}{pe}$$

k – number of categories (*k*=3, DOF=2)
n – forecast sample size
pe – expected probability = 0.33333
po – observed probability in FORECAST

EMPIRICAL PROBABILITY DISTRIBUTION

How to combine categories?

We fill in with BLUE, GREEN, or RED color

where maximum probability is featured by

BELOW, NEAR, or ABOVE normal category

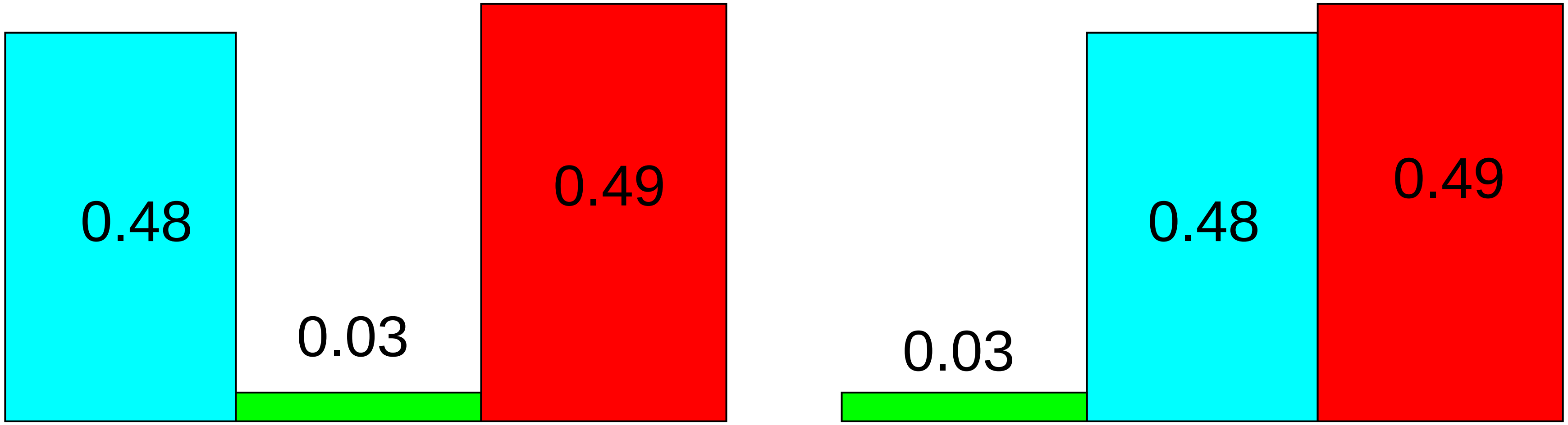
and distribution significantly differs from equiprobable.

If difference is not significant,

we fill in with WHITE color - UNCERTAINITY

EMPIRICAL PROBABILITY DISTRIBUTION

What to do if a histogram looks like below?



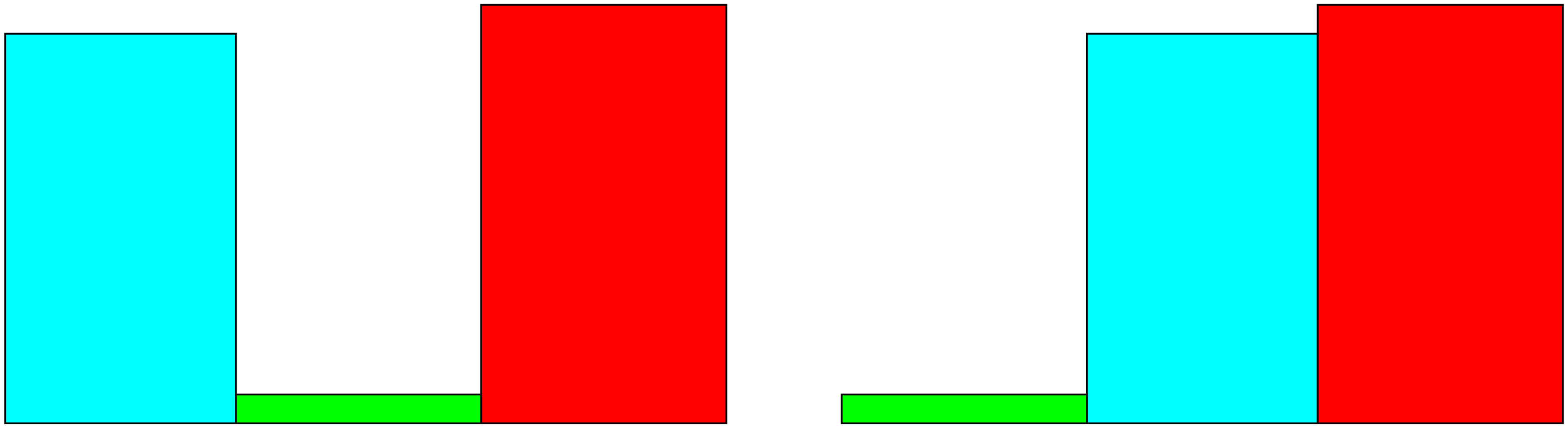
$$\chi^2 = n * 0.30$$

$$\chi^2_{threshold} = 5.99$$ (at the 5% significance level)

That is, when n>=20
there is a significant difference from equiprobable distribution

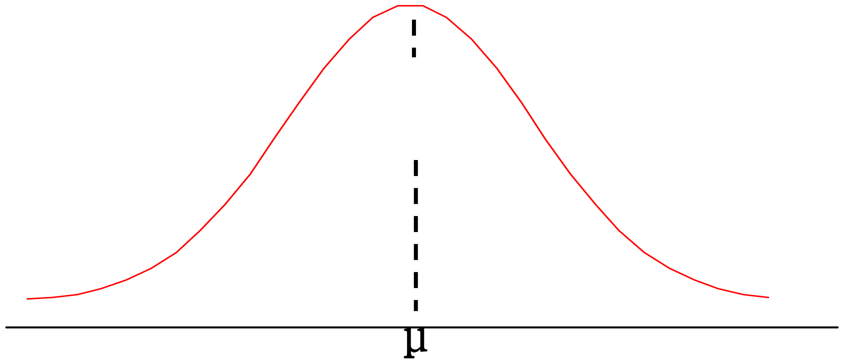
EMPIRICAL PROBABILITY DISTRIBUTION

Is there difference between these two distributions?



From the formal categorical point of view
there is no difference between shown distributions

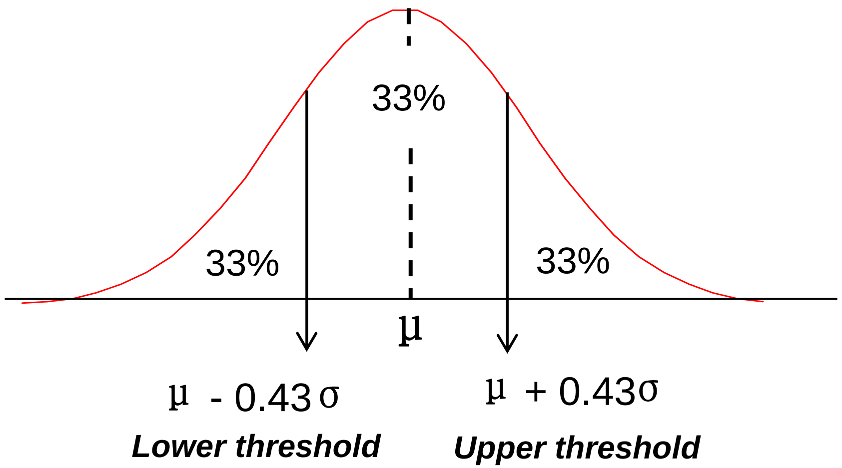
3. GAUSSIAN PROBABILITY DISTRIBUTION



$$p(x <= X) = \frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{X} \exp(-\frac{(x-\mu)^2}{2\sigma^2}) dx$$

Probability distribution of many environmental variables (Temperature, Pressure…) asymptotically converges to Gaussian N(μ,σ)

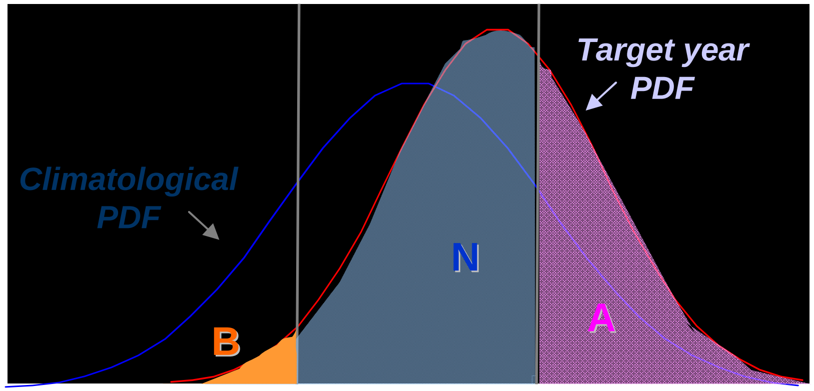
3. GAUSSIAN PROBABILITY DISTRIBUTION



So that, deviation from Gaussian distribution in hindcasts and forecast may be considered as errors due to small sample size.

In such a case we may define Xb and Xa as 33% and 67% quantiles of Gaussian Probability Distribution

3. GAUSSIAN PROBABILITY DISTRIBUTION



A : Probability of Above-normal
N : Probability of Normal
B : Probability of Below-normal

3. GAUSSIAN PROBABILITY DISTRIBUTION

combine categories

χ^2 distribution

$$\chi^2 = n\sum_{i=1}^{k}\frac{(pe - po)^2}{pe}$$

k – number of categories (*k*=3, DOF=2)
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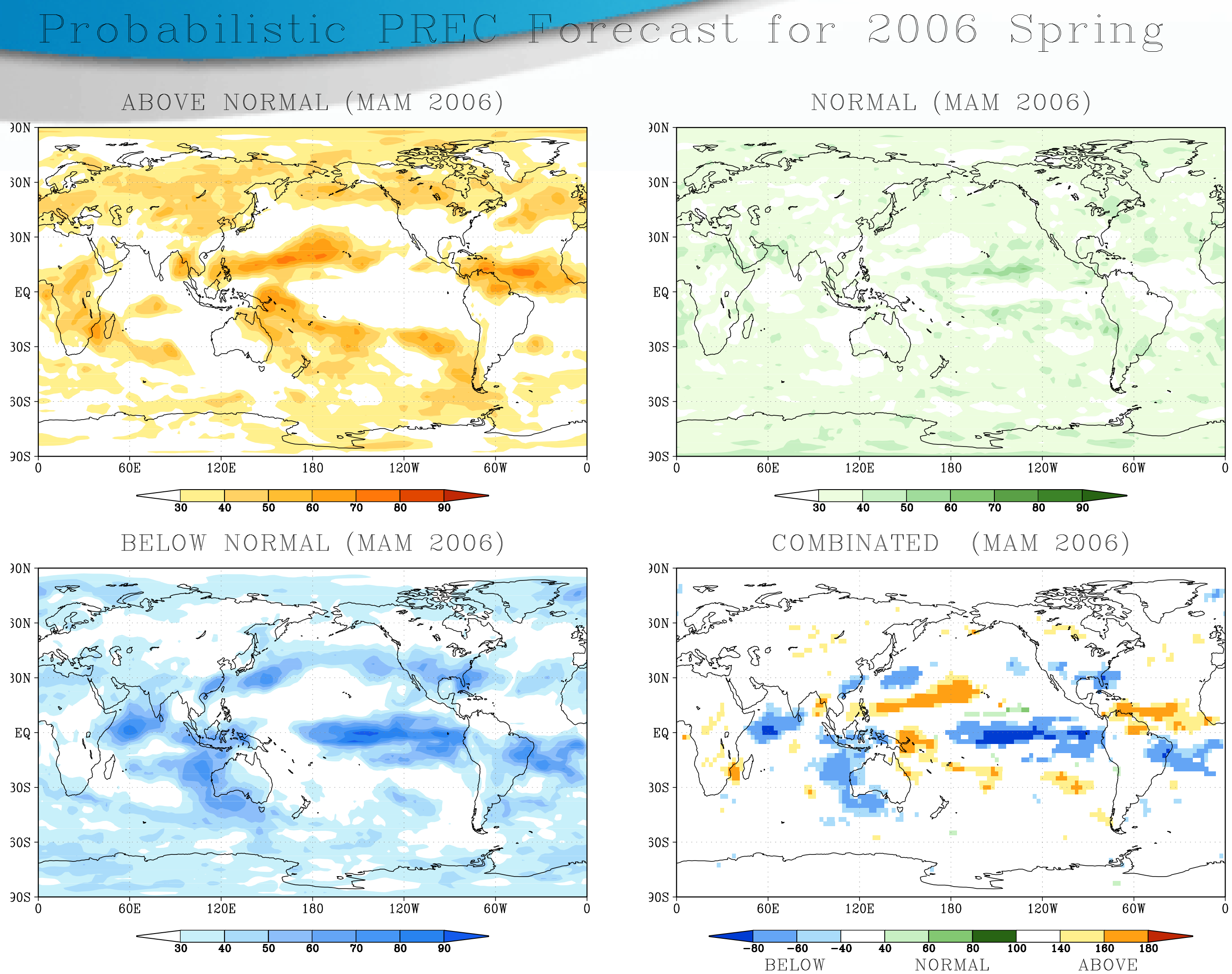


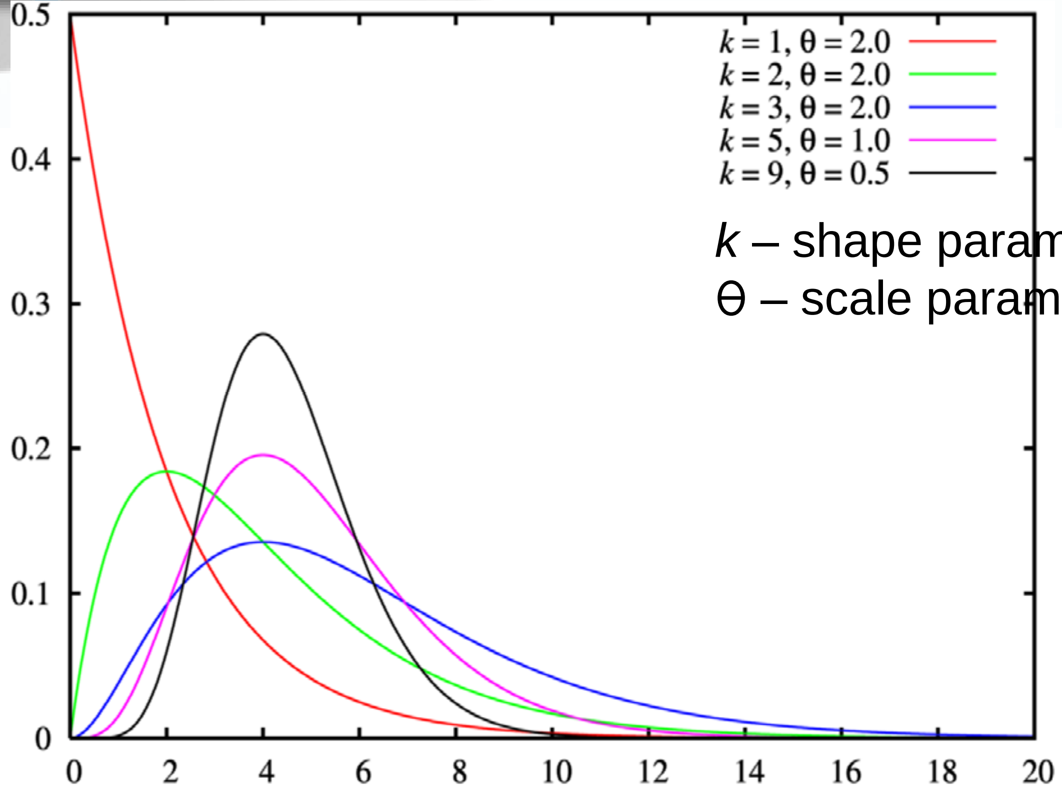
4. GAMMA PROBABILITY DISTRIBUTION

K. HA AND E. HA

Table II. Statistics for the monthly precipitation data of different epochs. The epochs were determined on the basis of Pettitt's change point detection test

Month	Epoch	N	Mean	Median	Standard deviation	Minimum	Maximum
April	1771–1886	116	102.6	96.5	63.4	6.0	322.0
	1887–2000	114	75.9	61.5	55.4	6.1	338.8
May	1771–1838	68	102.0	91.0	62.0	0.0	347.0
	1839–1881	43	140.7	118.0	76.8	28.0	420.0
	1882–2000	119	93.0	81.0	60.1	1.7	323.0
June	1771–2000	230	141.3	115.9	104.2	9.0	638.9
July	1771–2000	230	370.0	328.1	219.7	46.0	1426.0
August	1771–1778	8	137.6	163.5	73.3	46.0	228.0
	1779–1798	20	294.2	235.5	193.1	46.0	746.0
	1799–1877	79	371.5	338.0	214.4	38.0	1058.0
	1878–2000	123	270.9	234.0	184.2	13.4	1237.8
September	1771–1886	116	198.6	142.5	177.7	9.0	1236.0
	1887–2000	114	141.2	113.5	108.4	4.5	570.1
October	1771–2000	230	50.3	37.0	41.0	0.0	214.5
November	1771–1874	104	67.0	64.0	47.1	0.0	224
	1875–2000	126	48.0	36.3	47.1	0.0	443.8

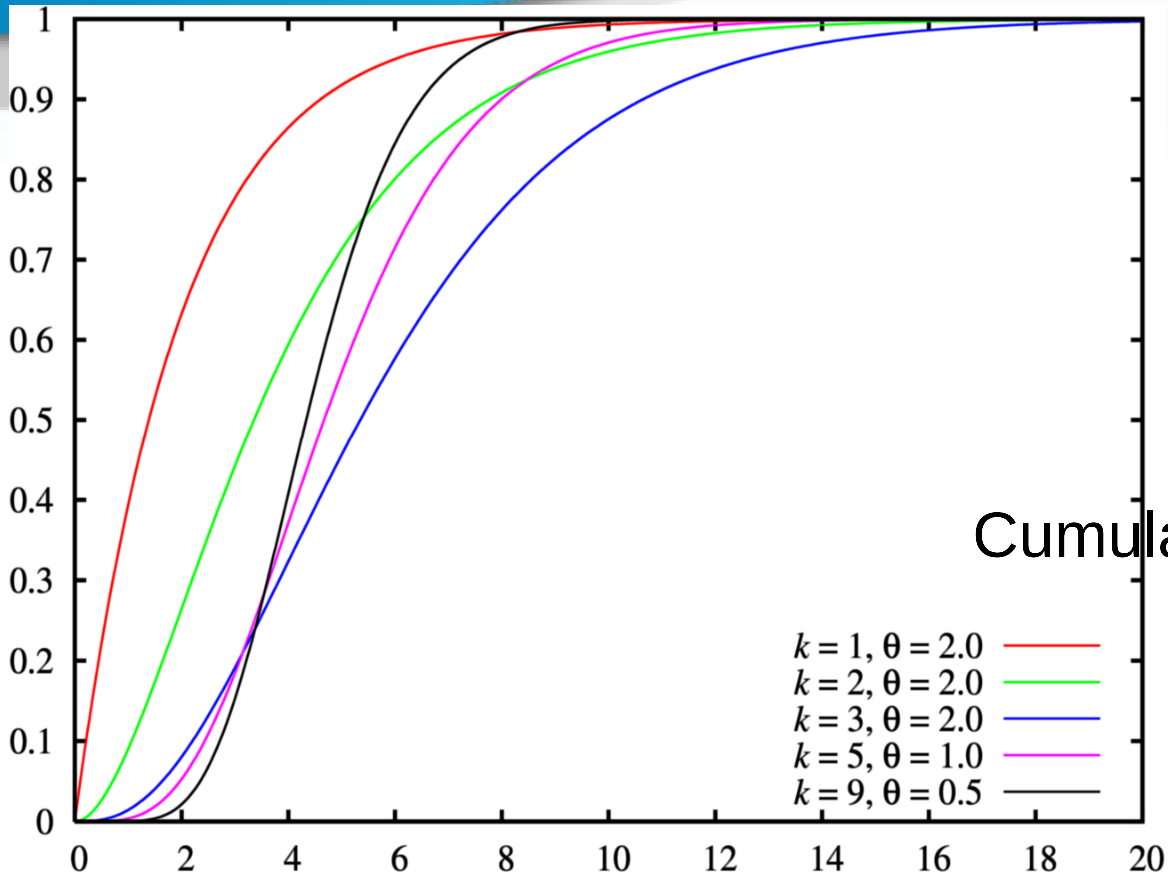
4. GAMMA PROBABILITY DISTRIBUTION



k – shape parameter
θ – scale parameter

Probability distribution of some environmental variables (Precipitation, Wind Velocity…) asymptotically converges to Gamma one

4. GAMMA PROBABILITY DISTRIBUTION



Cumulative PDF

Probability distribution of some environmental variables (Precipitation, Wind Velocity…) asymptotically converges to Gamma one

4. GAMMA PROBABILITY DISTRIBUTION

$$P(X \le x) = \frac{\int_0^X (x/\lambda)^{h-1} \exp(-x/\lambda)dx}{\lambda \int_0^\infty x^{h-1} \exp(-x)dx}$$

λ – scale parameter: $\hat{\lambda} = \hat{\sigma}^2 / \hat{\mu}$

h – shape parameter: $\hat{h} = (\hat{\mu} / \hat{\sigma})^2$

We are to find X_b *and* X_a :

$P(X_b)$ = 0.33 and $P(X_a)$ = 0.67 with estimated λ and h

4. GAMMA PROBABILITY DISTRIBUTION

$$P(X \le x) = \frac{\int_0^X (x/\lambda)^{h-1} \exp(-x/\lambda)dx}{\lambda \int_0^\infty x^{h-1} \exp(-x)dx}$$

Standardized Gamma cumulative distribution

new variable: $\beta = x/\lambda$

$\beta = \beta$[*i=i(h)*][*j = j(P=0.33* or *0.67)*] and *Xb;a=βb;aλ*

4. GAMMA PROBABILITY DISTRIBUTION

combine categories

χ^2 distribution

$$\chi^2 = n\sum_{i=1}^{k}\frac{(pe-po)^2}{pe}$$

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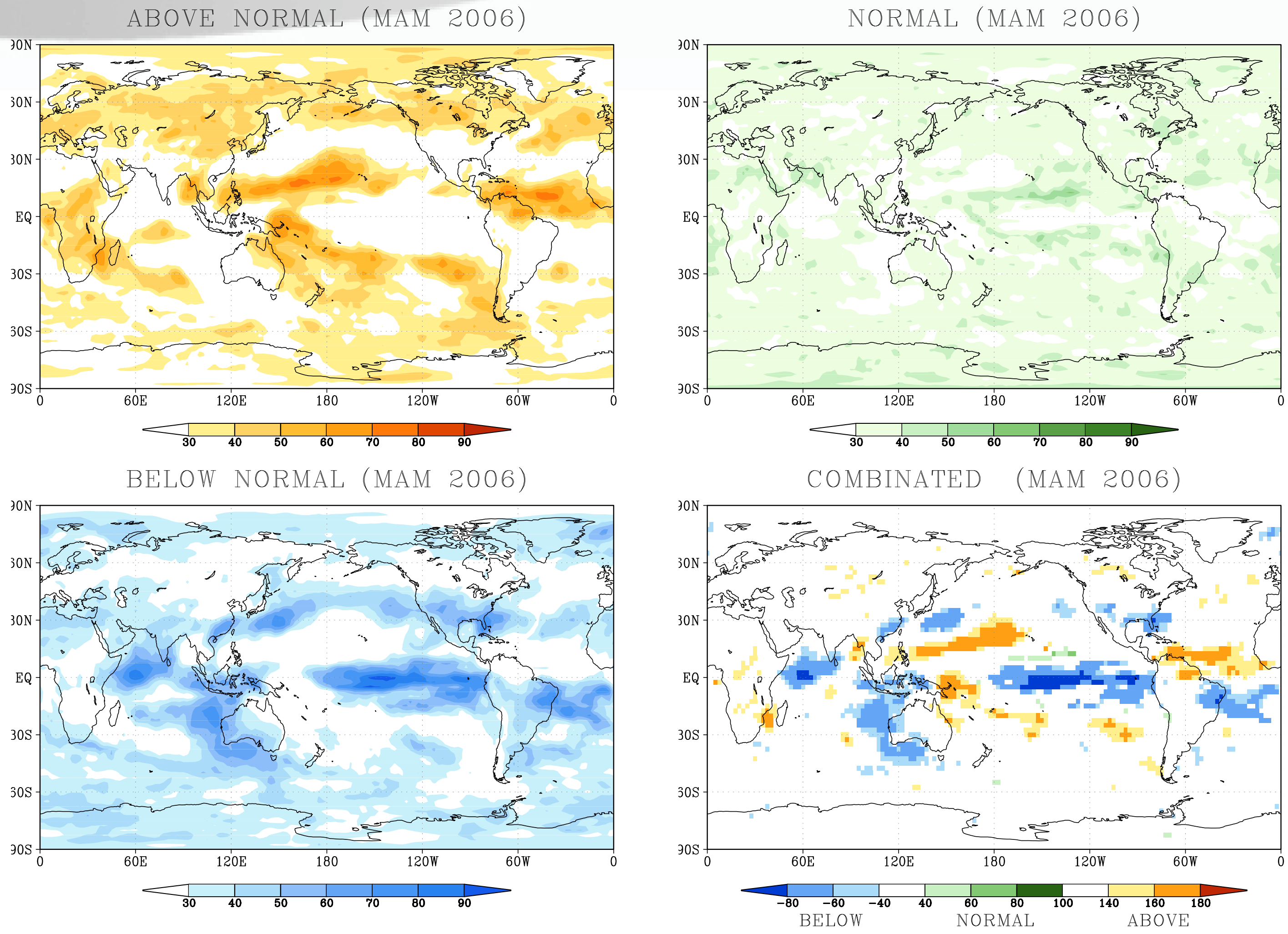
4. GAMMA PROBABILITY DISTRIBUTION

Probabilistic PREC Forecast for 2006 Spring



Advantages of approximation:
(both Gaussian and Gamma)

1. It is "technological" – well tabulated and easily computed

2. It smoothes sample errors

5. MULTI-MODEL ENSEMBLE

The rule of square root:

$$\sigma_{\mu} = \frac{\sigma}{\sqrt{N}}$$

Two marginal ways to combine multimodel ensemble

1. Estimate ensemble mean for each model separately, then process ensemble means (such scheme is usually used in deterministic forecast)

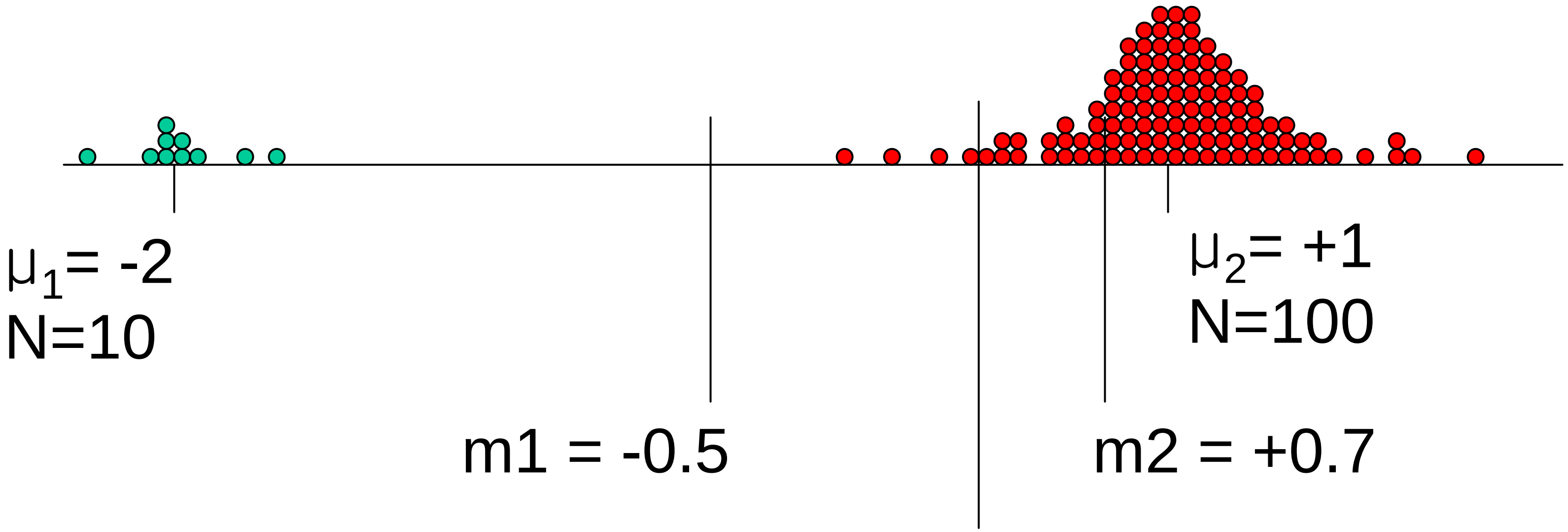
Shortcoming: it does not account for the number of ensemble members, i.e., it does not account to sampling error

2. Treat each ensemble member as independent realization. (such scheme sometimes is used in probabilistic forecast)

Shortcoming: models acquire the weights proportional to the number of ensemble members regardless of model quality

Two marginal ways to combine multi-model ensemble:

compare m1 and m2

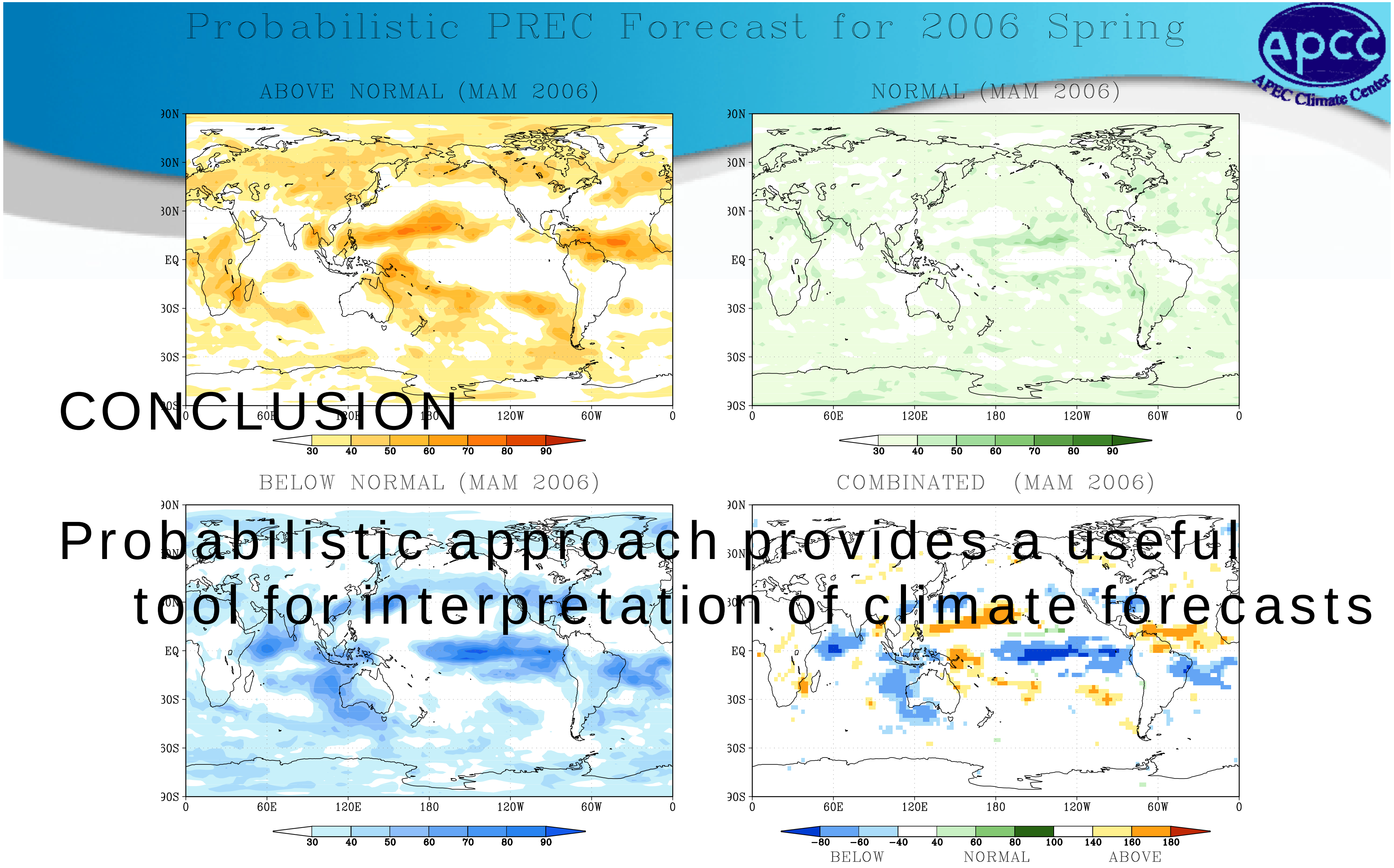


m = +0.3
(if to appoint the weight $N^{1/2}$)

CONCLUSION

1. Probabilistic approach provides a useful tool for climate forecasting

2. Probabilistic approach



CONCLUSION

Probabilistic approach provides a useful tool for interpretation of climate forecasts